Adiabatic geometric phases and response functions

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Treating a many-body Fermi system in terms of a single particle in a deforming mean-field, we relate adiabatic geometric phase to susceptibility for the non-cyclic case, and to its derivative for the cyclic case. Employing the semiclassical expression of the susceptibility, the expression for geometric phase for chaotic quantum system immediately follows. Exploiting the well-known association of the absorptive part of susceptibility with dissipation, our relations may provide a quantum mechanical origin of the damping of collective excitations in Fermi systems.

Chaotic adiabatic systems [1] serve as useful models for complex systems in the mean-field approximation. These idealized models have been employed to understand very interesting phenomena in nuclear [2] and plasma physics [3]. In particular, an important problem of many-body theory is to relate the slow collective excitations to faster single-particle motions. Adiabatic approximation leads us to linear response theory on one hand where dynamical susceptibility (or polarization propagator) [4] is central, and geometric phases on the other. A relation between these seemingly disparate quantities is being sought for in this Letter.

Phase factors of geometric origin were discovered for adiabatic quantum systems [5,6] and have been generalised to non-adiabatic situations [7]. The geometry of the Hilbert space plays a key role in understanding geometric phases [8], a recent illustration being for the Josephson junction [9]. Adiabatic geometric phase led to an understanding of several phenomena in physics [10] like fractional statistics in two-dimensional statistical mechanics, integer quantum Hall effect, anomalies in field theory, the Magnus force in the context of superfluidity [11] and so on. Recently, it has been shown that the viscosity of quantum Hall fluid in two dimensions at zero temperature is related to adiabatic curvature (whose flux gives the phase) [12]. An interesting aspect of many-body physics related to these advancements is in the Born-Oppenheimer approximation where reaction forces are shown to be of geometric origin [13]. Owing to an intimate tangle of collective and single-particle effects, it becomes useful to treat the total system as one where a slow subsystem is coupled to a faster one. In this setting, first-order corrections to the Born-Oppenheimer approximation leads to geometric magnetism and deterministic friction in a fully classical treatment, whereas a half-classical treatment gives rise to geometric magnetism only.

When a particle (a nucleon, say) moves inside an enclosure whose boundary is adiabatically vibrating in time, the wavefunction can acquire a geometric phase over a cycle of vibration. This model is an idealization of a single particle in a mean-field represented by the enclosure, and has been an established paradigm for numerous studies in the past [1]. We would particularly like to emphasize here the importance of such studies in enhancing the understanding of damping of collective excitations in nuclear physics. Concepts like time correlations and susceptibility are fundamental to any discussion of quantum statistical mechanics as they lead to an understanding of

the response of many-body system [4] and the transport coefficients [14]. It is well-known that the imaginary part of susceptibility is related to dissipation, thus it may be quite interesting to explore its possible relationship with a quantity like geometric phase. To work out such a relation, we begin with a discussion of geometric phase and identify the appropriate quantities related to susceptibility.

Let us first consider a Hamiltonian parametrised by $\mathbf{R}$ which describes a single particle in an effective mean-field where the shape of the field is vibrating adiabatically in time. It is well-known [5] that when the parameters evolve along a cyclic path, $\mathcal{C}$, the instantaneous eigenfunction of the system, $|n(\mathbf{R})\rangle$ corresponding to the eigenvalue $E_n(\mathbf{R})$, acquires a geometric phase given by

$$\gamma_n(\mathcal{C}) = \oint_{\mathcal{C}} i\langle n(\mathbf{R}|\nabla_{\mathbf{R}}n(\mathbf{R})\rangle . d\mathbf{R} = -\frac{1}{\hbar}\int_{\mathcal{S}}\mathbf{V}_n . d\mathbf{S}, \quad (1)$$

where $\mathcal{S}$ is the surface enclosed by $\mathcal{C}$ in the parameter space, and $\mathbf{V}_n$ is the "field-strength" (adiabatic curvature) given by a familiar expression involving a sum of weighted wedge product between two appropriate matrix elements :

$$\mathbf{V}_n = -i\hbar \sum_{m(\neq n)} \frac{\langle n|\nabla_{\mathbf{R}}H|m\rangle \wedge \langle n|\nabla_{\mathbf{R}}H|m\rangle}{(E_n - E_m)^2}. \quad (2)$$

A suitable form of $\mathbf{V}_n$ for the sequel is [15]

$$\mathbf{V}_n = \frac{i}{2\hbar}\lim_{\epsilon \to 0}\int_0^{\infty}dt e^{-\epsilon t}t\langle n|\left[(\nabla_{\mathbf{R}}H)_t, \wedge(\nabla_{\mathbf{R}}H)\right]|n\rangle \quad (3)$$

where $(\nabla_{\mathbf{R}}H)_t$ denotes the Heisenberg-evolved operator. Note that, the state $|n(\mathbf{R})\rangle$ appearing in (3) corresponds to a single-particle eigenket in an effective mean-field. This state is clearly related to the original many-body Fermi system for which the imaginary part of the dynamical susceptibility is [14]

$$\chi^{"}(t) = \frac{1}{2\hbar}\langle \Phi_0|[\hat{\mathcal{A}}(t), \hat{\mathcal{B}}(0)]|\Phi_0\rangle$$
$$= \int \frac{d\omega}{2\pi}e^{-i\omega t}\tilde{\chi}^{"}(\omega) \quad (4)$$

where $|\Phi_0\rangle$ is the pure ground state of the many-particle system with Fermi energy, E_F. If one-body operators, $\hat{H}$, $\hat{A}$, and $\hat{B}$, are used to construct many-body operators by a direct sum so as to get $\hat{\mathcal{H}}$, $\hat{\mathcal{A}}$, and $\hat{\mathcal{B}}$, respectively, and $\hat{\mathcal{H}}|\Phi_l\rangle = \mathcal{E}_l|\Phi_l\rangle$, then we [16] have

$$\langle \Phi_0|\hat{\mathcal{A}}|\Phi_l\rangle = \langle m(\mathbf{R})|\hat{A}|n(\mathbf{R})\rangle. \quad (5)$$

On reducing the many-body system at $T = 0K$, (where the Fermi-Dirac distribution is a Heaviside step function), to one-body system, we can express [16]

$$\tilde{\chi}^{"}_{AB}(\omega) = -\frac{\omega}{2}\int dt e^{i\omega t}\,\text{tr}\,\delta(E_F - H)[\hat{A}(t), \hat{B}(0)]. \quad (6)$$

This can also be written semiclassically as [16]

$$\tilde{\chi}^{"}_{AB}(\omega) = -\frac{\omega}{2}\int dt \int \frac{d^f x d^f p}{(2\pi\hbar)^f}\delta(E - H)A_W(\mathbf{x}, \mathbf{p})$$
$$\left[e^{(i\omega - \hat{\mathcal{L}}_{cl})t}B_W\right](\mathbf{x}, \mathbf{p}) + \mathcal{O}(\hbar^{-f+1})$$
$$- \frac{\omega}{2\pi\hbar}\delta(E_F - H)\sum_{p,r}\frac{\cos\left(\frac{r}{\hbar}S_p - r\frac{\pi}{2}\nu_p\right)}{|\det(\mathbf{m}_p^r - \mathbf{I})|^{1/2}}.$$
$$. \int dt e^{i\omega t}\oint_p d\tau A_W(\tau)B_W(\tau + t) + \mathcal{O}(\hbar^0) \quad (7)$$

where the subscript $'W'$ refers to the Weyl symbol of the operator, f denotes the degrees of freedom and $\hat{\mathcal{L}}_{cl}$ is the Liouvillian operator. The last term corresponds to the periodic orbit expansion where S_p, ν_p, and $\mathbf{m}_p$ correspond to action, Maslov index, and the monodromy matrix for the periodic orbit, p, and r denotes the repetitions. We have used the Gutzwiller trace formula [17] for the case where the single-particle motion is chaotic. The above semiclassical expression is valid for Hamiltonian operators which are quadratic in momentum, $\hat{p}$, with an additive position-dependent term. The expression (6), however, is general. For the case where the system has partially broken symmetry, the results have been recently generalized [18].

The label 'n' in (3) corresponds to single-particle states and is related to $|\Phi_0\rangle$ because the many-body matrix elements can be written in terms of one-body matrix elements for the case when all the constituents are taken as non-interacting. In many-body physics, this gives the zero-order response whereupon the interaction can be included in a Vlasov description in an iterative way [19]. For relating the response function to the geometric phase, the operators $\hat{\mathcal{A}}$ and $\hat{\mathcal{B}}$ in our discussion are to be identified with $\nabla_X\hat{\mathcal{H}}$ and $\nabla_Y\hat{\mathcal{H}}$ for $\mathbf{R} = (X, Y, Z)$.

The matrix element in (3) can be written as a many-body matrix element using (5) by composing $\hat{A}$ and $\hat{B}$ so that we get the operator, $\mathcal{C}(t) = [\mathcal{A}(t), \mathcal{B}(0)] - [\mathcal{B}(t), \mathcal{A}(0)]$, which s related to a difference $\chi^{"}_{AB}(t) - \chi^{"}_{BA}(t) = \chi^{"}_C(t)$. Thus, we can re-write $\mathbf{V}_n$ as

$$\mathbf{V}_n = \frac{i}{2\hbar}\lim_{\epsilon \to 0}\int_0^{\infty}dt e^{-\epsilon t}\,t\,\langle \Phi_0|\mathcal{C}|\Phi_0\rangle$$
$$= \int_0^{\infty}dt\,t\chi^{"}_C(t) = -\frac{\partial\tilde{\chi}^{"}(\omega)}{\partial\omega}\bigg|_{\omega=0}. \quad (8)$$

We now arrive at our first relation for the case of cyclic evolution :

$$\gamma_n(\mathcal{C}) = \int_{\mathcal{S}}d\mathbf{S}.\frac{\partial\tilde{\chi}^{"}_C(\omega; \mathbf{R})}{\partial\omega}\bigg|_{\omega=0}. \quad (9)$$

Since $\tilde{\chi}^{"}_C(\omega)$ is related to energy dissipation, this relation connects geometric phase to dissipation.

Exploiting (7), we get

$$\left.\frac{\partial \tilde{\chi}^{"}_{AB}(\omega;\mathbf{R})}{\partial \omega}\right|_{\omega=0} = -\frac{1}{2}\int dt \int \frac{d^f x d^f p}{(2\pi\hbar)^f}\delta(E_F - H)$$

$$[A_W\left(e^{-\mathcal{L}_{cl}t}B_W\right) - B_W\left(e^{-\mathcal{L}_{cl}t}A_W\right)]$$

$$-\frac{1}{2\pi\hbar}\lim_{\omega\to0}\delta(E_F - H)\sum_{p,r}\frac{\cos\left(\frac{r}{\hbar}S_p - r\frac{\pi}{2}\nu_p\right)}{|\det(\mathbf{m}_p^r - \mathbf{I})|^{1/2}}$$

$$\int dt e^{i\omega t}\oint_p d\tau\,[A_W(\tau)B_W(\tau+t) - B_W(\tau)A_W(\tau+t)]\quad(10)$$

which entails the semiclassical expression for adiabatic geometric phase for chaotic systems, using (9). The first term comes from the classical time correlation function which is expected to decay exponentially as the dynamics is chaotic. This decay is governed by the largest Liapunov exponent of the classical system. The second term is the semiclassical correction in terms of periodic orbits which can be termed as a periodic orbit two-form. It is important to note that (9) is valid for general Hamiltonians whereas (10) is restricted to the operators like $\hat{p}^2 + V(\hat{q})$ only.

The time-dependent deformation of the mean-field can, in general, be non-cyclic, particularly due to the fact that several harmonics (possibly incommensurate) may be involved. In this general case, the above expression does not hold and we now proceed to quantify the general relation.

When a quantal system does not follow a cyclic evolution the geometric phase not only depends on the path of the evolution curve but also on the end points. It has been shown in [20] that the wavefunction of the system can acquire a geometric phase during its time evolution, irrespective of adiabatic and cyclic condition, which is given by

$$\gamma(\Gamma) = \int i\langle\chi(t)|\dot{\chi}(t)\rangle dt,\quad(11)$$

where $|\chi(t)\rangle$ is a modified state vector defined from the actual state vector $|\Psi(t)\rangle$ of the system as $|\chi(t)\rangle = \frac{\langle\Psi(t)|\Psi(0)\rangle}{|\langle\Psi(t)|\Psi(0)\rangle|}|\Psi(t)\rangle$ and an overdot denotes the time-derivative. From this general expression, the adiabatic, open-path geometric phase can be written in the following way,

$$\gamma_n(\Gamma) = \int_\Gamma i\langle\chi_n(\mathbf{R})|\nabla\chi_n(\mathbf{R})\rangle.d\mathbf{R} = \int_\Gamma \Omega_n(\mathbf{R}).d\mathbf{R},\quad(12)$$

where $\Omega_n(\mathbf{R})$ is a generalised adiabatic vector potential (connection one-form) whose line integral gives the geometric phase. This non-cyclic adiabatic geometric phase can be expressed as a line integral of the difference of two vector potentials [21],

$$\gamma_n(\Gamma) = \int_\Gamma(\mathbf{A}_n(\mathbf{R}) - \mathbf{P}_n(\mathbf{R})).d\mathbf{R},\quad(13)$$

where $\mathbf{A}_n(\mathbf{R}) = i\langle n(\mathbf{R})|\nabla n(\mathbf{R})\rangle$ is the usual Berry potential whose curl gives the adiabatic curvature $\mathbf{V}_n$ and

$\mathbf{P}_n(\mathbf{R}) = -\Im(\frac{\langle n(\mathbf{R}(0))|\nabla n(\mathbf{R})\rangle}{\langle n(\mathbf{R}(0))|n(\mathbf{R})\rangle})$ is an extra potential that takes care of the contributions from the end points of the evolution path. The non-cyclic geometric phase is gauge invariant because under a local gauge transformation, $\mathbf{A}_n$ and $\mathbf{P}_n$ transform in the same way. Also in the special case of cyclic evolutions of parameters, (12) goes over to the well known expression, (1).

For our purpose, we simplify the generalised vector potential, $\Omega_n(\mathbf{R})$, as

$$\Omega_n(\mathbf{R}) = \mathbf{A}_n(\mathbf{R}) - \mathbf{P}_n(\mathbf{R})$$
$$= \Im\sum_{m(\neq n)}\frac{\langle n(\mathbf{R}(0))|m(\mathbf{R})\rangle}{\langle n(\mathbf{R}(0))|n(\mathbf{R})\rangle}\frac{\langle m(\mathbf{R})|\nabla H|n(\mathbf{R})\rangle}{(E_n - E_m)}.\quad(14)$$

Using an integral representation of $(E_n - E_m)^{-1}$, we can write $\Omega_n(\mathbf{R})$ as follows

$$\Omega_n(\mathbf{R}) = \frac{1}{\hbar}\sum_{m(\neq n)}\lim_{\epsilon\to0}\int_0^\infty dt e^{-\epsilon t}\Im\left[ie^{-i(E_n - E_m)t/\hbar}\right.$$
$$\left.\frac{\langle n(\mathbf{R})|P_n(\mathbf{R}(0))P_m(\mathbf{R})\nabla H|n(\mathbf{R})\rangle}{|\langle n(\mathbf{R}(0))|n(\mathbf{R})\rangle|^2}\right]\quad(15)$$

where $P_n(\mathbf{R}(0)) = |n(\mathbf{R}(0))\rangle\langle n(\mathbf{R}(0))|$ and $P_m(\mathbf{R}) = |m(\mathbf{R})\rangle\langle m(\mathbf{R})|$ are the projection operators corresponding to nth and mth eigenstates. Now define a quantum correlation function between the operator, A, and the Heisenberg evolved operator, B_t, through

$$Q_{AB}(t) = \frac{1}{2}\langle n|(AB_t + B_t A)|n\rangle - \langle n|A|n\rangle\langle n|B_t|n\rangle\quad(16)$$

and identify the Hermitian operators $A = P_n(\mathbf{R}(0))$ and $B = \nabla H$. With this, the generalised vector potential can be written in terms of a quantum correlation function:

$$\Omega_n(\mathbf{R}) = \frac{1}{\hbar}\lim_{\epsilon\to0}\int_0^\infty dt e^{-\epsilon t}\frac{Q_{AB}(t)}{|\langle n(\mathbf{R}(0))|n(\mathbf{R})\rangle|^2}\quad(17)$$

Let us note that $Q_{AB}(t)$ defines fluctuations in the symmetrised correlation of two operators. Recalling the relationship between the Fourier transform of $Q_{AB}(t)$ and the susceptibility $\tilde{\chi}"(\omega)$ [22] at zero temperature, we find

$$\tilde{Q}_{AB}(\omega) = \hbar\tilde{\chi}"(\omega),\quad(18)$$

an instance of the fluctuation-dissipation theorem of the second kind [23]. Following the physical arguments used to express single particle expectation values in terms of statistical quantities as in the cyclic case, we get

$$\tilde{\chi}"(0;\mathbf{R}) = \frac{1}{\hbar}\lim_{\epsilon\to0}\int_0^\infty dt e^{-\epsilon t}Q'_{AB}(t)$$
$$= \Omega_n(\mathbf{R}),\quad(19)$$

where $Q'_{AB}(t) = \frac{Q_{AB}(t)}{|\langle n(\mathbf{R}(0))|n(\mathbf{R})\rangle|^2}$ is the scaled quantum correlation function. Here, the scale factor is the survival probability $P(t) = |\langle n(\mathbf{R}(0))|n(\mathbf{R})\rangle|^2$ which is related to

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the two-time correlation function at zero temperature for a system in canonical equilibrium [24]. This result seems to be a fundamental one as the generalised adiabatic vector potential is exactly equal to the imaginary part of the susceptibility at zero frequency. Therefore, the adiabatic geometric phase for a non-cyclic variation of external parameters along a path Γ is given by

$$\gamma_n(\Gamma) = \int_\Gamma \tilde{\chi}''(0; \mathbf{R}).\mathbf{dR}, \qquad (20)$$

which relates a bulk property of the many-body Fermionic, chaotic quantum system to a physical quantity assigned to a single particle.

In (9) and (20), note that the frequency-dependent response functions are different. However, they are related as follows : for a cyclic evolution along a path, $\mathcal{C}$, applying Stokes' theorem, (20) reduces to a surface integral of $\mathrm{curl}\tilde{\chi}''(0; \mathbf{R})$ over $\mathcal{S}$. Using (9), it follows that

$$\nabla_\mathbf{R} \times \tilde{\chi}''(0; \mathbf{R}) = \left. \frac{\partial \tilde{\chi}''_C(\omega; \mathbf{R})}{\partial \omega} \right|_{\omega=0}. \qquad (21)$$

The above results have clear relevance to finite Fermi systems like metallic clusters and nuclei. The historical collective picture of nucleus was Bohr's liquid drop which has been successfully used ever since. Due to the numerical simulations [1] and the results (eqs.(9) and (20)) obtained above, it is clear that the nuclear fluid has, in fact, characteristics of a gel which is an elastic or a viscoelastic liquid. The connecting relation (21) between cyclic and non-cyclic case is necessary as a consistency requirement. The applicability of our results to nuclei is due to the compelling evidence, originating from shell structure of binding energies, testifying that the mean free path of nucleons is larger than the nuclear dimensions [25]. Hence it is natural to look for the source of viscosity in the nuclear fluid in terms of single-particle features where the effective mean-field leads to chaotic dynamics.

It is well known that viscosity or friction in a quantal context can appear in thermodynamic limit. Since we have consistently discussed many-body Fermi systems in equilibrium, the seemingly contradictory conditions of finiteness of the system and "continuity" of the energy spectrum are met with. The finite-size of the system explicitly manifests itself in terms of the sum over periodic orbits. Thus, yet another way to look at our relations (9), (20) is that these relations are off-springs of finite-size and spectral continuity- a stylized version of thermodynamic limit where viscosity and geometric phase can be discussed together.

The notion of viscoelasticity in the context of nuclear physics has been pointed out in Ref. [1]. If we accept that the model for nucleus is a gel model and apply linear viscoelastic theory [26] where stress relaxation function is connected to the time-correlation function, we can see that the generalised vector potential (connection one-form) is directly related to viscosity. This is due to the fact that the time integral of the relaxation function is identified with viscosity in linear viscoelastic theory.

Every time some relationships with possible fundamental relevance are given, the question about their generality arises. The relation (9) and (20), so-to-say (γ_n-χ) relations, are restricted by the limits $T \to 0$. For any non-zero T, the semiclassical result for susceptibility is already known [16]. We believe that the (γ_n-χ) relation will be given as above where $\tilde{\chi}''$ will be evaluated at some non-zero value of ω consistent with the temperature. Hence, as long as the dynamics is chaotic, these relations hold. This takes us to a deep interplay of quantal and statistical features embodied in the subject of quantum chaos. It is interesting to mention here that recent works have led to a new treatment of quantum statistical mechanics in terms of Berry conjecture [27,28]. Further, it has been shown [29] that thermal fluctuations for large number of degrees of freedom coincide with quantum fluctuations for chaotic quantum systems in the semiclassical limit when the freedoms are smaller in number. One can immediately rationalise above discussion in the light of these contemporary works. In the above discussion, when we pick up a single-particle state, it ought to be one that can be written as a random superposition of plane waves in the sense of [27]. This is possible for a system with chaotic classical dynamics. It is these states which can be combined into a Slater determinant leading to the Fermi-Dirac distribution.

In conclusion we have established a fundamental relation between adiabatic geometric phase and response function for cyclic as well as non-cyclic evolutions of chaotic quantum systems where chaos appears as a result of the mean-field seen by a single-particle due to other particles. Since the imaginary part of the susceptibility is related to energy dissipation, our result provides a quantum mechanical way to interpret and understand decay of collective excitations of Fermi systems. The results of this Letter can be generalised for the case when the dynamics is not fully chaotic using the results of [17].

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